

High temperature superconductivity: from complexity to simplicity

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I discuss the recent quantum oscillation experiments in the underdoped high temperature superconductors.

HAMLET: Do you see yonder cloud that's almost in shape of a camel ?

POLONIUS: By th' mass and 'tis, like a camel indeed.

HAMLET: Methinks it is like a weasel.

POLONIUS: It is backed like a weasel.

HAMLET: Or like a whale.

POLONIUS: Very like a whale.

—*William Shakespeare*

More than 20 years ago, Bednorz and Müller discovered superconductivity in copper oxides at remarkably high temperatures [1]. Since then, physicists have struggled to understand the mechanisms at work. Recently, a set of experiments on cuprates in high magnetic fields [2, 3, 4, 5, 6] has completely changed the landscape of research in high-temperature superconductors (HTSs). In particular, the data suggest that the current carriers are both electrons and holes, when in fact the materials are hole doped—i.e., the current carriers should be positively charged. Moreover, the data cannot be reconciled with an important theorem about how electrons are organized in materials [7] unless one assumes that the signals arise from a combination of both holes and electrons. Until now, physicists have not been able to decide whether the cuprates, in Shakespeare's terms, are camels or whales; in fact, these experiments foreshadow a remarkable degree of simplicity in these complex materials.

The cuprates start out as insulators and become superconductors when doped with additional charge carriers. These so-called Mott insulators insulate by virtue of strong repulsive Coulomb interaction and need not break any symmetries in the lowest energy state, the ground state. A symmetry of a system is a transformation, such as a translation or a rotation, that keeps it unchanged. Such a symmetry is said to be broken, or spontaneously broken, if the system does not obey the symmetry of the underlying fundamental physical nature of the material; for example, a ferromagnet breaks the spin-rotational symmetry with its magnetization pointing in a definite direction. The notion of symmetry and broken symmetry finds many deep applications in physics.

Soon after the discovery of the cuprate superconductors, Anderson proposed [8] that their parent compounds begin as a featureless spin liquid that does not break any symmetries, called the resonating valence bond (RVB) state: "The preexisting magnetic singlet pairs of the insulating state become charged superconducting pairs when

the insulator is doped sufficiently strongly" [8]. Unfortunately, experiments show that the insulating phase is a simple antiferromagnet in which the spins are arranged in antiparallel manner, that is, with a broken symmetry. The materials remain antiferromagnets for a range of doping, and then, after a sequence of not well understood states as a function of doping, they become superconductors.

How this plays out experimentally can be understood by looking at the Fermi surface, a fundamental concept in condensed matter physics. The Fermi surface differentiates the occupied electronic states from the unoccupied states (in coordinates of momentum rather than real space). Electrons fill the Fermi surface (FS) up to some highest occupied energy called the Fermi energy (see the figure). The excitations from the FS (e.g., when a current flows) are called Landau quasiparticles (quasi, because they are combinations or superpositions of real particles). The robustness of FS is due to its topological invariance, one of the most basic invariances in mathematical physics, which signifies stability with respect to "small deformations" [9]. Even when the quasiparticles are absent due to electron-electron interactions, as in one-dimensional electronic systems, the FS is still defined by the same topological invariant. A reconstruction of this surface, such as a break up of a single surface into hole-like and electron-like pockets, requires a global deformation, most likely a broken symmetry. The new experimental work (26) yielded measurements of the oscillations that arise from energy levels created by imposing a magnetic field on the material (the Landau levels). As the magnetic field is increased, the highest fully occupied levels sweep past the Fermi energy, and the system periodically returns to itself, hence the oscillation in physical properties. The oscillations of the Hall resistance [2, 4], capable of detecting the sign of the charge carriers, seem to show the presence of electron and hole pockets in the Fermi surface, suggesting that it undergoes some kind of reconstruction.

One might complain that these high field measurements are still considerably below the upper critical field where superconductivity disappears (about 100 T or more) and are affected by the complex motion of vortices generated by the magnetic field. This may be true, but quantum oscillations in many superconductors are observed at fields as small as half the critical field, with the oscillation frequencies unchanged from the nonsuper-

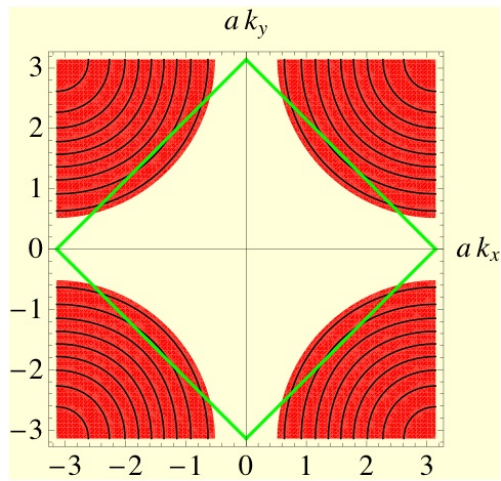
conducting state (with an increased damping, however). It is also known that the quasiparticles of HTSs do not form Landau levels (9). Thus, it is very likely that the quantum oscillation experiments are accessing the normal state beyond the realm of superconductivity. But what kind of state? As the oscillations definitively point to both electron and hole pockets, it cannot be a conventional Fermi surface, rather one that has undergone a reconstruction due to a broken symmetry at variance with the RVB picture [11].

We may be finally beginning to understand these superconductors after two decades. The fly in the ointment is the lack of observation of electron and hole pockets in other measurements in hole-doped superconductors (in angle resolved photoemission spectroscopy, for instance) that are also capable of measuring Fermi surfaces [see, however, the work on electron-doped materials [12]]. Missing so far in experiments are also the higher frequency oscillations that must arise from the hole pockets, not just the electron pockets [4]. With further experimental work, we should be able to tell just what kind of animal we are dealing with. [Note added: A higher frequency is now seen and is believed to arise from an incommensurate order—S. Sebastian, N. Harrison, and G. Lonzarich, private communication.]

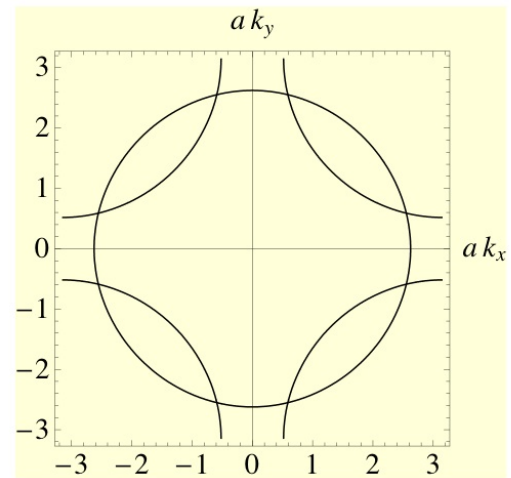
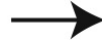
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FIG. 1: Fermi surface reconstruction: In a crystalline lattice of periodicity a , the available quantum states are contained within the Brillouin zone (BZ). In (a) we have shown a two-dimensional example, $-\frac{\pi}{a} \leq k_x < \frac{\pi}{a}$, $-\frac{\pi}{a} \leq k_y < \frac{\pi}{a}$, which is pertinent to high temperature superconductors that consist of weakly coupled CuO-planes. For N -atoms, with unit cells containing 1 atom in a square $a \times a$, there are exactly $2N$ available states, as each state can be filled by electrons with two distinct spin orientations. The diamond in (a) is called the reduced Brillouin zone (RBZ) and contains exactly half the number of available states. In (a) the unoccupied states are colored red; these, holes, in a otherwise filled BZ, respond as charge carriers with positive sign in response to electric fields. The constant energy contours are shown as the set of black curves. The filled diamond corresponds to one electron per unit cell. The complementary sets within the BZ, when reassembled together, will form an identical diamond, which we may say is filled with holes. In (a) the red area corresponds to $(1+x)$ holes per unit cell of the crystal lattice. The excess, x , is called the doped holes. High temperature superconductors are schizophrenic. In some regimes they behave as though they consist of $(1+x)$ charge carriers, holes, while in the underdoped regime their properties are determined instead by x doped holes. This is an important mystery. A class of theories posit that FS reconstructs in the underdoped regime. Consider shifting the FS in (a) by vectors $(\pm\frac{\pi}{a}, \pm\frac{\pi}{a})$, which will give rise to the Figure (b), ignoring the shading for clarity. Interesting quantum mechanical processes, about which there can be much debate, can result in reconstructions shown in (c), as in a kaleidoscope. However, if we continued to consider the full BZ, we would double the number of states. All distinct states are contained in the RBZ, but there are now two distinct sets of energy levels, the upper band and the lower band. However, we continue to use the full BZ in (c), as a better aid for visualization. Because the RBZ is the fundamental unit in the wave vector space, the new unit cell of the crystal lattice is doubled, given by a square $\sqrt{2}a \times \sqrt{2}a$, and the full translational symmetry of the original lattice is broken. The FS now consists of *disconnected sheets* of blue and red areas. The remarkable fact is that the charge carriers in the blue region behave like electrons of fraction n_e and in the red region like holes of fraction n_h . The doped holes is easily seen to be $x = 2n_h - n_e$, as there are two hole pockets and one electron pocket in the RBZ. The broken symmetry invoked here is called commensurate, as the translational invariance of the crystal of integer multiples of the next nearest neighbor lattice vectors of the original lattice is still preserved. The broken symmetry can also be incommensurate with the original crystal lattice and can give rise to more complex FS reconstructions.

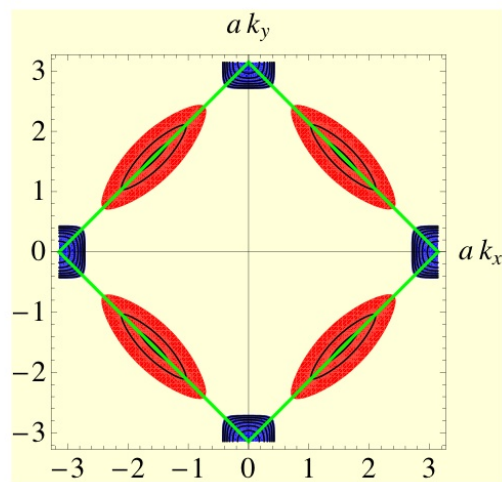
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(a)



(b)



(c)